Newton's Method for Finding Roots

A laboratory exercise—Part III

Newton's method is a very good "root finder." When it works, it works very well and converges quickly to the root of the function. But sometimes, Newton's method doesn't work so well. In this portion of the lab, you will explore the limitations of Newton's method and try to draw some conclusions about how to use it "wisely."

III. How good is Newton's method?

As you have seen in the last several problems, it is very fast (lots of accuracy with few iterations) if you choose your initial guess wisely. This is the short answer to the question. However, a poor choice for a first guess can sometimes lead to difficulties. You will explore a few possibilities in the next several problems.

10. Suppose any one of the "guesses" at the root is a place where the derivative of the function is zero. Explain, both analytically and graphically, why the method "crashes."

Retrieve the Maple program newtanim.mw from the P:\ data drive. Familiarize yourself with the various components of the program.

11. Suppose we want to solve the equation $-x^3 + 8x^2 - 80 = 0$. Start with a nice graph of the function to get an "eyeball estimate" of the solution to $-x^3 + 8x^2 - 80 = 0$.

    (For this problem, set the "window" in newtanim.mw to the intervals $[-12, 10] \times [-200, 200]$.)

    (a) Let $x_0 = 4.7$ Have the the Maple program newanim.mw show you the first twenty iterations of Newton's method on the graph of f. What happens? Why? In this case, does Newton's method ever find the solution? If so, how many iterations does it take, approximately?

    (b) Let $x_0 = 5.35$. Have the the Maple program newanim.mw show you the first six iterations of Newton's method on the graph of f. What happens? Why?

    (c) Use the Maple program and an initial guess of $x_0 = 0$. What happens? Why?

12. Try applying Newton's method to the function $f(x) = \dfrac{x}{1+x^2}$ with $x_0 = 6$. What happens? Why does it happen? (For this problem, you will want to set your window to positive x-values and to go fairly far out to the right.)

13. Try applying Newton's method to the function $f(x) = x^3 - x$.

    (a) Compute the first 3 iterations of Newton's method starting with an initial guess of $x_0 = \dfrac{1}{\sqrt{5}}$. Set up the recursion relation and do the algebra. No decimal approximations! (Feel free to use Maple to help you, but don't use Newt.mw or Newtanim.mw) What happens?

    (b) Use newtanim.mw to draw the first **three** iterations of Newton's method for the previous part. (Does this help to illuminate the algebraic solution?)

    (c) Let $x_0 = .55$. Have the the Maple program newanim.mw show you the first ten iterations of Newton's method on the graph of f. What happens? Why?

Final question: Despite the various weird scenarios that we have explored above, Newton's method is really very easy to use effectively. The problems in this last section were designed to show you what sorts of things to avoid when making an initial guess. Given your romp through the "dark side" of Newton's method, what kinds of rules of thumb could you come up with for finding an initial guess that isn't going to get you into trouble?